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# Hybrid markets, tick size and investor trading costs ☆

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**Abstract**

This paper shows how the tick size affects equilibrium outcomes in a hybrid stock market such as the NYSE that features both a specialist and a limit order book. Reducing the tick size facilitates the specialist's ability to step ahead of the limit order book, resulting in a reduction in the cumulative depth of the limit order book at prices above the minimum tick. If market demand is price-sensitive, and there are costs of limit order submission, the limit order book can be destroyed by tick sizes that are either too small or too large. We show that trading cost is minimized at larger tick sizes for larger market orders, creating an incentive to submit smaller orders when tick size is reduced. With a smaller tick size, specialist participation increases and specialist profit increases slightly for small market orders, and considerably for large market orders.
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## 0. Introduction

On January 29, 2001, the NYSE completed its shift to decimalization. The SEC had mandated the shift, saying that not only would it be easier for investors to understand trading, but it would make stock prices "more competitive."

Today, it seems clear that decimalization has not been equally beneficial to everyone. There is now an extensive empirical literature studying how the move from eighths to

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sixteenths and then to pennies affected trading costs for both retail and institutional traders. On a positive note, many studies report an overall reduction in quoted and effective spreads, which reduced trading costs for small traders,[1] although large traders seem to benefit less from spread reduction (Bacidore et al., 2003, 2001; Ronen and Weaver, 2001). Recently, a United States Government Accountability Office Report (2005) found that trading costs fell for both retail and institutional traders.

The news does not appear to be all good, however. Goldstein and Kavajecz (2000) report that the move to sixteenths reduced cumulative depths throughout the limit order book on the NYSE, making large traders worse off. Jones and Lipson (2001) also report that while the cost of trading a small order was little affected by the move, trading a large order became more expensive, and that the cost differential increased with order size. They report that overall, the move to sixteenths increased trading costs. Following decimalization, there was a massive 66% decline in the cumulative depth in the limit order book (Research Division of the NYSE). Reflecting that reduction in depth, Bollen and Busse (2005) find that trading costs for actively managed mutual funds increased by a remarkable 1.367 percent of fund assets. Likely responding to that reduction in depth, Chakravarty et al. (2003) find that institutional traders re-allocated order flow toward electronic networks; the United States Government Accountability Office Report (2005) found that large traders now split their orders into smaller components, despite the associated fixed costs of doing so. Indeed, average trade size on the NYSE in 2004 was 67% smaller than in 1999.

Another indication that decimalization has raised trader costs is a Charles Schwab's report of a 22% increase in cancellations or changes of limit orders in the five days following the NYSE's completion of decimalization (AP February 12, 2001): traders have to monitor their orders more carefully. Bacidore et al. (2003) find a reduction in limit order size and an increase in limit order cancellation rate, as well as a reduction in displayed liquidity further from the quote midpoint. Ronen and Weaver (2001) do not document a reduction in quoted depth; however, they hypothesize that the specialist convention of quoting minimum depth may be masking a reduction in public limit depth. Perhaps most surprisingly, a careful analysis by Chakravarty et al. (2003) reveals that decimalization significantly reduced not only trading volume, but even the total number of trades.[2] Chakravarty et al. (2001) find that the change in trading volume is not uniform across order sizes: while small size trades and trading volume increased, large size trades and volume decreased.

An understanding of market design is crucial for unraveling why the reduction in tick size was not uniformly beneficial. The NYSE is a hybrid market in which a market order can be crossed against both a limit order book and a specialist/floor broker. On the NYSE, limit orders are submitted before a market order is realized, and accordingly have priority at the same price over the specialist or competing floor brokers. Given the incoming market order and the limit order book, the specialist or a floor broker can choose whether to undercut with a slightly better price any portion of the book and take the remainder of the trade. The penny tick dramatically reduced the cost of stepping ahead of limit orders, providing specialists and floor brokers a significant advantage at the expense of other traders. The consequences for limit orders are summarized by the complaint about the

[1] See, e.g., Bollen and Whaley (1998), Goldstein and Kavajecz (2000), Jones and Lipson (2001), Chakravarty et al. (2001), Bessembinder (2003), Smith et al. (2005).

[2] Chou and Lee (2003) also find that volume per trade decreased significantly after decimalization.

impact of decimalization by institutional traders that "their efforts to buy large blocks of stock on the market are being blocked by specialists who 'step in' at the last minute and bid a penny higher to buy stocks that institutional investors would have gotten otherwise" (AP (February 17, 2001) report). As long as submitting limit orders is either directly costly or indirectly costly because limit orders can become stale due to information arrival—and a specialist can selectively step in front of limit orders—then to offset the reduced likelihood of execution, the optimal response of limit traders may be to submit fewer orders and set prices further from the quote mid-point.[3] The contribution of this paper is to explore this optimal response. That is, we investigate how the hybrid market design of the NYSE interacts with the tick size to affect limit order depth, specialist profits, trading costs, and trade sizes.

Rock (1990) was the first to model a hybrid market structure. Our model is similar to that in Seppi (1997), except in our model, market demand is price-sensitive. First, competitive liquidity providers submit limit orders. Next, a price-sensitive market trader receives a liquidity shock and places a market order. Finally, the specialist decides which portion of the limit order book to undercut. In equilibrium, limit order traders break even conditional on their orders being executed—the (exogenous) cost of order submission equals their (positive) expected trading profits.

We find that in equilibrium, as in Seppi (1997), at every price save the smallest, the cumulative depth of the limit order book falls as tick size is reduced, because the specialist finds undercutting more attractive. Because of the price-sensitive market demand, unlike in Seppi (1997), market order size is endogenous. This endogenous reduction in market demand reinforces the direct effect on depth, as the reduced market demand further reduces the value of submitting a limit order. Indeed, as in Glosten and Milgrom (1985), when market demand is too price-sensitive, the limit order book can become empty. We show that fixing tick size, an increase in price sensitivity of market orders reduces depth.

We then show that *fixing* price-sensitivity, a reduction in tick size reduces depth. When market demand is sufficiently price-sensitive, markets feature a non-empty limit order book only when the tick size is sufficiently large. For smaller tick sizes, the increased ability of the specialist to undercut the limit book makes it impossible for limit order traders to break even at any price. The equilibrium limit order book is empty, and consequently, there are wide quoted spreads. Parlour and Seppi (2003) recognize that a competing limit order market can cause similar problems for a hybrid market.[4]

The endogenous reduction in market demand suggests that the specialist might prefer a large tick size. However, we show that the specialist prefers the smallest tick at which the limit order book is not empty, which in our view is most of the time. That is, the contribution to specialist profits from the increased ease of undercutting exceeds the

[3] On a related note, Harris (1996) argues that a smaller tick makes it more profitable for front-runners to take trades away from large traders in markets that enforce time priority. Consistent with this argument, he finds that order display decreases as tick size falls.

[4] A similar type of market failure can occur in the environment of Foucault et al. (2003), who also consider a limit market and show that the effect of tick size on the speed of spread improvement after transaction interacts with the proportion of patient to impatient traders. With discreteness, the patient traders improve on the spreads by more than they would if the tick were zero. With many impatient traders, the cost of waiting for order execution is small, and the patient traders are reluctant to improve on the existing spreads. When the proportion of impatient traders is large enough, a sufficiently small tick size eliminates any spread improvement, and the resulting spread is very wide with zero depth at intermediate ticks, the situation referred to as "pathological."

decrease in profits due to the endogenous reduction in market demand. Importantly, the increase in specialist profits following tick size reduction is larger when market orders are large. Our results are consistent with the evidence in Coughenour and Harris (2004) that specialist profits and participation rates increased after the decimalization for stocks in which public order precedence used to be especially costly (small stocks and actively traded stocks with tight spreads). For large stocks, they find specialist profits to decline. The reduction in profits for the larger stocks may have resulted from the splitting of large orders into small components. In fact, in our model the specialist's share would fall to zero if all orders were sufficiently small.

Because limit order traders break even, average market trader losses rise as tick size falls. It is complicated to determine how the price paid by each *individual* market order is affected when tick size falls—some pay less and others pay more. The reason is that although the specialist pursues a more aggressive undercutting strategy, limit depth also declines, so that a greater portion of the market order is filled by the specialist. Our numerical simulations confirm that investors who submit small (large) orders benefit from smaller (larger) tick sizes. Interpreting this result in the context of tick size reduction on the NYSE, the greatest beneficiaries were small retail traders as opposed to large institutional traders (see, e.g., Goldstein and Kavajecz, 2000; Jones and Lipson, 2001; Bollen and Busse, 2005). We also show that traders who wish to submit large market orders reduce their order size as tick size gets sufficiently small, and hence, the tick size maximizing average trade size on a hybrid market is positive.

Note that our price-sensitivity parameter resembles the impatience parameter in Parlour (1998) or Rosu (2005): lower price sensitivity may be thought of as higher impatience. However, unlike Parlour (1998), Foucault et al. (2003) or Rosu (2005), traders in our model do not choose between providing and supplying liquidity; rather they remain in separate groups and choose whether or not to trade. Thus higher impatience, or lower price-sensitivity of market order traders, increases market demand and simultaneously induces higher limit order supply at every price. In contrast, in a model like that in Foucault et al. (2003), higher impatience does not increase limit order supply; instead, it moves limit order supply away from the quote midpoint.

Finally, we discuss the empirical implications of our analysis. An important prediction of our model is that a reduction in tick size increases the cost of submitting a large order. Albeit outside of our formal analysis, this prediction implies that as tick sizes fell on the NYSE, order splitting should have become more common. Order splitting would lead to both a smaller average trade size and a smaller dispersion in the distribution of order sizes on the NYSE. Further, it might cause specialist participation rates and profits to fall. The preponderance of empirical evidence suggests that after decimalization on the NYSE, average trade size fell, specialist participation rates fell, and specialist profits fell. However, Coughenour and Harris (2004) find that specialist profits and participation rates increased for some stocks. We suggest that cross-sectionally, the effect of decimalization on specialist profits and participation rates should be related to the increase in order splitting due to decimalization. For stocks where the smaller tick did not cause order splitting to become more prevalent (i.e., stocks where the average trade size was small to begin with), specialist participation rates and profits should increase. For stocks where average trade size decreased substantially, specialist profits and participation rates should not increase much, and might perhaps fall.

The paper is organized as follows. We next present and analyze the model. We characterize equilibrium outcomes in Section 2. In Section 3 we analyze via a numerical example the effects of tick size on aggregate volume and on trading costs. In Section 4, we provide empirical implications for trading volume and the distribution of order sizes as a function of tick size on both a hybrid market and on a pure limit order market. Section 5 concludes. Proofs to Propositions 1 and 2 are in the appendix. Proofs to Propositions 3 and 4 are available upon request.

## 1. The model

Our model builds on Seppi (1997). There is an asset that pays out $v$ per share. This value is common knowledge. Agents can submit market orders to a hybrid limit order/specialist market. The limit market is made by a continuum of agents with preferences $c_1 + qv$, where $c_1$ is consumption of money and $q$ is the number of shares of the asset held. Market orders can also be handled by a specialist who shares the preferences $c_1 + qv$.

We focus on the market buy orders and limit sell orders so that we consider market order submitters with a relative preference for the asset, with preferences $c_1 + q\beta v$, where $\beta > 1$.[5] We assume that $\beta$ is distributed according to $G(\cdot)$ with density $g(\cdot)$. After Proposition 2, for ease of exposition we let $\beta$ be uniformly distributed on $[\beta_{\min}, \beta_{\max}]$. Fixing $\beta_{\min}$, as we increase $\beta_{\max}$, the preference for the asset relative to cash increases, making market order demand less price sensitive. That is, the greater is $\beta$, the less price sensitive are market orders. Seppi (1997) considers the case where market demand is price-insensitive, $\beta = \infty$. We normalize investors' initial endowments to zero. There is a distribution $F(\cdot)$ over the maximum number of shares, $N$, that a liquidity trader can buy, where $N$ and $\beta$ are independently distributed.

The liquidity trader can buy claims to the asset at prices, $p_j$, that are positive integer multiples of a tick, $d > 0$. Hence, if $v$ is also an integer multiple of $d$, then $p_j = v + jd$, $j = 1, \ldots, \infty$. Otherwise, $p_j = v + (j-1)d + \varepsilon$, where $0 < \varepsilon < d$. At each price, liquidity providers who submit limit orders at that price have priority over the specialist. For liquidity providers other than the specialist, there is a small per-share cost $c > 0$ of submitting a limit order, which is less than the tick size, i.e., $c < d$.[6] The specialist can trade costlessly. Finally, as in Seppi (1997), there is a trading crowd with a reservation price $r$ that can absorb arbitrarily large orders. Alternatively, one could assume that there is no trading crowd, but that the specialist never quotes a price greater than $X\%$ above $v$. This closely mirrors the price continuity requirement on the NYSE that prevents price-gouging. This alternative assumption leaves the qualitative results unaffected.

The market timing is as follows:

1. Liquidity providers submit limit orders. We denote the depth at price $p_j$ by $s_j$.
2. A liquidity trader is selected and submits her market order.
3. The specialist offers a price, deciding which limit orders, if any, to undercut.
4. Trades are consummated and payoffs are realized.

[5]The analysis of market sell orders is analogous.

[6]In a previous draft, we allowed the limit order submission cost to be endogenous by changing the timing slightly to allow limit orders to become stale with some probability. The submission cost was the expected loss that the stale limit order incurred when it was picked off by the specialist.

To simplify the analysis, we initially assume that the common asset valuation, $v$, and the crowd's reservation price, $r$, are multiples of the tick size, $d$. Later, we provide the analysis of trade sizes and trading costs that relaxes this assumption.

## 2. Equilibrium

Let $p(\cdot)$ be the price schedule faced by the liquidity trader. Given $\beta$, $N$, the market trader will buy $M$ shares, $M = \min(N, Y)$, where $Y$ is the greatest number such that $\beta \geqslant p(Y)/v$. Liquidity providers other than the specialist submit limit orders so that the marginal limit trader at each price earns zero expected profits.

Integrating over possible types $(\beta, N)$, we compute the expected profits to each limit order. The zero-profit limit order at $p_j$ solves $Pr(executed) \times (p_j - v) = c$. Finally, the specialist chooses a clean-up price to maximize profits: he undercuts the limit order book at the price that maximizes trading profits. We assume the specialist undercuts the book if indifferent.

The specialist's expected profit from undercutting price $p_j$ is

$$\mathrm{E}[\pi_{j-1}] = (p_{j-1} - v)\left(M - \sum_{i=1}^{j-1} s_i\right).$$

The specialist's expected profit from not undercutting is:

$$\mathrm{E}[\pi_j] = (p_j - v)\left(M - \sum_{i=1}^{j} s_i\right).$$

He therefore undercuts when

$$\mathrm{E}[\pi_{j-1}] = (p_{j-1} - v)\left(M - \sum_{i=1}^{j-1} s_i\right) \geqslant \mathrm{E}[\pi_j] = (p_j - v)\left(M - \sum_{i=1}^{j} s_i\right). \tag{1}$$

Given that the common valuation $v$ is on the grid, we can substitute $v + jd$ for $p_j$, so that equation (1) simplifies: the specialist undercuts price $p_j$ if and only if

$$M \leqslant \sum_{i=1}^{j-1} s_i + js_j. \tag{2}$$

Let $t_j$ be the maximum market order size such that the specialist undercuts price $p_j$:

$$t_j = \sum_{i=1}^{j-1} s_i + js_j. \tag{3}$$

In turn, this implies that a liquidity trader faces cut-off price $p_j$ if and only if

$$t_j < M \leqslant t_{j+1}. \tag{4}$$

Substituting Eq. (3) into (4) we see that inequality (4) holds if and only if

$$s_{j+1} > \frac{j-1}{j+1} s_j. \tag{5}$$

If condition (5) is violated, then it cannot be an "equilibrium." To see this, observe that if condition (5) is violated, then it is never optimal for the specialist to quote $p_j$; he does better to quote $p_{j+1}$. That is, the specialist's "clean-up" price jumps from $p_{j-1}$ to $p_{j+1}$. But then, execution probabilities at $p_j$ and $p_{j+1}$ are the same because limit orders at $p_j$ are executed only when those at $p_{j+1}$ are. The only time that limit orders at $p_j$ are undercut by the specialist is when those at $p_{j+1}$ are also undercut. Because per-share revenues differ at the two prices, marginal limit order submitters cannot be indifferent between them, and hence it cannot be an equilibrium.

Seppi (1997, Proposition 2) shows that when market order flow is price insensitive, then in equilibrium, $s_{j+1}$ is always large enough relative to $s_j$ that (5) holds. We will show that (5) holds independently of tick size if market orders are sufficiently price-insensitive, and for more price-sensitive market demand only if the tick size is sufficiently large.

When is a limit order at price $p_j$ executed? First, the liquidity trader is willing to pay the price: $\beta v > p_j$. Second, the liquidity shock must be large enough: $N > t_j$. Next, note that at every price exceeding $p_1$, either all limit orders at a particular price are executed, or none are. This follows because the specialist's undercutting threshold, $t_j$, exceeds the cumulative depth in the limit order book up to price $p_j$. Hence, if the market order exceeds $t_j$, then no limit orders at price $p_j$ are undercut. Otherwise, all are. The sole exception is when the asset value is not on the grid so that $p_1 - v$ is less than a full tick. Then, the specialist cannot profitably undercut $p_1$.[7] The probability of execution at $p_j$ is

$$Pr(execution) = Pr(N > t_j) Pr\left(\beta > \frac{p_j}{v}\right),$$

and the zero-profit/indifference condition for the marginal limit order at $p_j$ is

$$(1 - F(t_j))\left(1 - G\left(\frac{p_j}{v}\right)\right) = \frac{c}{p_j - v} = \frac{c}{jd}. \tag{6}$$

Using (6), we solve recursively for the limit order book at each price. At $p_1$,

$$(1 - F(s_1))\left(1 - G\left(\frac{p_1}{v}\right)\right) = \frac{c}{d},$$

and

$$s_1 = \begin{cases} H\left(\dfrac{c}{d\left(1 - G\left(\frac{p_1}{v}\right)\right)}\right); & \dfrac{c}{d(1 - G(\frac{p_1}{v}))} \leqslant 1, 0; \text{otherwise.} \end{cases} \tag{7}$$

At the next price, the threshold is $t_2 = s_1 + 2s_2$. Unless this threshold is exceeded, a limit order at price $p_2$ is not executed. Substituting the solution for $s_1$ into the corresponding zero-profit condition, we can solve for $s_2$; and continuing we can solve for the entire book.

The following three propositions characterize the equilibrium and the effect of tick size and price sensitivity of market demand on the equilibrium, *provided* that the limit order book is not empty. In Proposition 1 we prove uniqueness. In Proposition 2, we establish that for a given distribution for $\beta$, decreasing the tick size both causes the limit order book

[7] The zero-profit condition at $p_1$ is not altered by this assumption, because for the marginal limit order trader, the probability of execution is still the probability that the market order is enough to fill the depth at $p_1$. If either $p_1$ is not affordable, or the liquidity shock is too small, the marginal limit order at $p_1$ is not executed, so we compute the depth at $p_1$ in the same way regardless of whether the specialist finds it profitable to undercut $p_1$ or not. At prices above $p_1$, all limit traders are marginal since either all limit orders at a particular price are executed, or none are. At $p_1$, unlike at higher prices, if $v$ is not on the grid, not all limit orders are marginal.

to thin out and increases expected specialist profits. In Proposition 3, we show that as the price sensitivity of market demand increases, aggregate depth falls at every price. We next deal with the interaction between tick size and price sensitivity. In particular, in Proposition 4, we show that if market demand is sufficiently price-sensitive, the equilibrium limit order book will be empty when the tick size is sufficiently small.

**Proposition 1.** *If the limit order book is non-empty, the equilibrium is unique and can be found using* Seppi's (1997) *solution procedure.*

**Proposition 2.** *Provided that the equilibrium limit order book is non-empty, the cumulative depth $Q_j = \sum_{i=1}^{j-1} s_i$ at or below any price $p_j$ on grids $P$ such that $p_j \in P$ decreases as tick size decreases, and the specialist's expected profit increases as tick size decreases, regardless of the price sensitivity of market demand.*

**Proposition 3.** *Let $\beta$ be uniformly distributed over $[\beta_{\min}, \beta_{\max}]$. If the equilibrium limit order book is non-empty, increasing price sensitivity by reducing $\beta_{\max}$ reduces depth at every price, provided that the probability density, $f(\cdot) = F'(\cdot)$ over the maximum number of claims, $N$, does not increase "too" steeply in $N$, i.e., $f(t_j)$ is not too large relative to $f(t_{j-1})$ so that*

$$-(j-1)f(t_{j-1})\{(\beta_{\max}-p_j)^2(\beta_{\min}-p_j)+(\beta_{\min}-p_j)(2d(\beta_{\max}-p_j)+d^2)\}$$
$$+jf(t_j)\{(\beta_{\max}-p_j)^2(\beta_{\min}-p_j)+d(\beta_{\max}-p_j)^2\}>0.$$

*If $N$ is uniformly distributed, the above condition holds whenever the book is non-empty.*

The restriction on the distribution over the maximum number of claims that a liquidity agent would purchase is reasonable. In fact, the most reasonable parameterization of $f(\cdot)$ is that it is nonincreasing, i.e., larger orders are less likely.

**Proposition 4.** *Let $\beta$ be uniformly distributed over $[\beta_{\min}, \beta_{\max}]$. Suppose that market demand is sufficiently price-sensitive in the sense that $\beta_{\max} < 2r/v - 1$. Then there exists a critical tick size, $d^* = \frac{1}{3}(2(r-v)-(\beta_{\max}-1)v)$, such that for all $d < d^*$, the limit order book is empty.*

## 3. The effect of tick size on trading costs and trade sizes

In Section 2, we showed that market liquidity falls when tick size decreases. We now show that different types of liquidity traders are differentially affected by changes in tick size: those who submit small orders prefer small tick sizes, and those who want to trade in volume prefer larger tick sizes.

In our environment, both market order size and trading costs are affected by tick size. This follows because market order size is not exogenous. Therefore, different traders who happen to trade $M$ when the tick size is $d$ buy many different amounts when the tick size is $d'$. Consequently, they are likely to disagree about which tick size is best. We next compute the trading costs for small, medium, and large orders in order to determine which tick size minimizes these trading costs. Consider an example with $\beta \sim U[1,5]$, $N \sim U[0,10]$, the true value, $v = 1+\eta$, $\eta$ small, the crowd reservation value, $r = 3$, the limit order submission cost, $c = \frac{1}{33}$. We calculate the total trading cost (that includes trading against the limit order book as well as trading the remainder of the order, if any, against the specialist at the clean-up price) for different order sizes from $M = 0.5$ to $M = 10$ for different tick sizes, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, and $\frac{1}{32}$. Figs. 1 and 2 show the representative results. The main message is that the trading cost associated with submitting a market order is minimized at different tick sizes

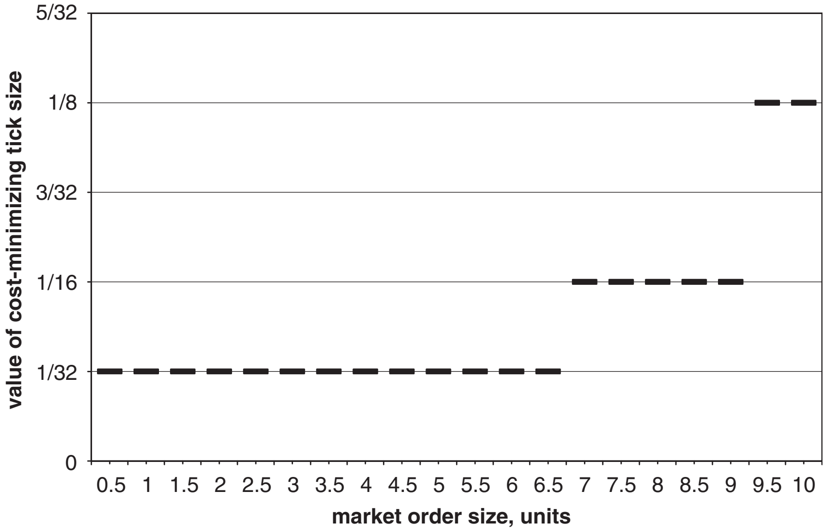


Fig. 1. The figure shows the value of tick size that minimizes the cost of trading a market order of a given size. The cost is measured as the total dollar cost of trading through the limit order book, up to the specialist's clean-up price, plus the cost of trading the remainder (if any) of the order against the specialist at the clean-up price. Size of market order is in units. The figure is based on numerical simulations with the following parameters: asset value $v = 1 + \eta, \eta$ small; crowd reservation price $r = 3$; price elasticity of market demand, β, is uniform on [1, 5]; liquidity shock, N, is uniform on [0,10]; per-unit cost of submitting a limit order, $c = \frac{1}{33}$.

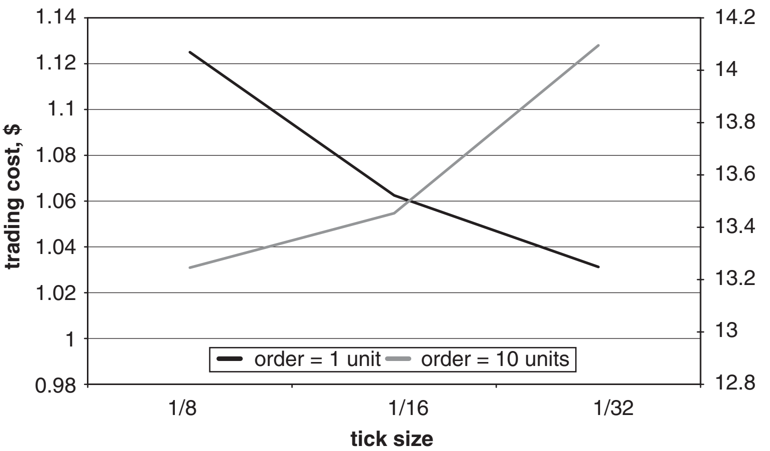


Fig. 2. The figure shows the dollar cost of trading a market order of a given size (left scale: cost of trading one unit; right scale: cost of trading ten units), under different values of tick size. The cost is measured as the total dollar cost of trading through the limit order book, up to the specialist's clean-up price, plus the cost of trading the remainder (if any) of the order against the specialist at the clean-up price. Size of market order is in units. The figure is based on numerical simulations with the following parameters: asset value $v = 1 + \eta, \eta$ small; crowd reservation price $r = 3$; price elasticity of market demand, β, is uniform on [1, 5]; liquidity shock, N, is uniform on [0,10]; per-unit cost of submitting a limit order, $c = \frac{1}{33}$.

for different order sizes. The cost of submitting a small, intermediate, and large market order is minimized at a tick size of $\frac{1}{32}$, $\frac{1}{16}$, and $\frac{1}{8}$, respectively. The result is qualitatively similar under various parameter specifications.

This is not surprising. Traders who submit small orders trade only at the lowest price in the book, and thus benefit from a small tick size. Traders who submit larger orders benefit more when depth in the book at higher prices is greater, which is the case when the tick size is larger. Figs. 1 and 2 illustrate this tradeoff, and show that for large orders, adequate depth is more important than receiving a very good price on a small portion of one's order.

We next investigate what happens to trade sizes as tick size falls. That is, we investigate how trader types who used to submit small (large) orders under a larger tick size change their trade size under a new smaller tick size. Our model implies two countervailing effects of reducing tick size on the average trade size. On the one hand, reducing tick size enables highly price-sensitive traders to enter the market so that their trade size increases. On the other hand, those traders who are already submitting large orders under the larger tick size have an incentive to reduce their order size as tick size falls. To see why, observe that from (6), at any fixed price $p_j$, the threshold market order $t_j$ is independent of tick size. Consider a trader with highest affordable price $p'$. If the tick is $d$, the next available price is $p' + d = p''$. The corresponding threshold amounts are $t'$ and $t''$. Suppose $N$ is large enough to exceed $t''$. Under the tick size $d$, she can submit an order as large as $t''$ and still trade at her highest affordable price $p'$ since the specialist will undercut $p''$ unless the market order exceeds $t''$. Consider now what happens if the tick size is reduced to $\frac{d}{2}$. The threshold $t^*$ at the new intermediate price $p^*$ satisfies the condition $t' < t^* < t''$, since (i) $t'$ and $t''$ do not change with tick size, and (ii) thresholds are monotonically increasing in price in equilibrium. Now, to ensure that the specialist still trades at $p'$, the trader must not submit an order larger than the new intermediate threshold, which is smaller than $t''$. Hence, trading volume falls with tick size. What happens if $N$ is not sufficient to exceed $t''$? In that case, as tick size falls, the market order will either stay the same size if $N \leqslant t^*$, or decrease if $N > t^*$. Finally, what happens if the highest affordable price is $p^*$? In that case, reducing tick size does not affect trading volume because the same threshold $t''$ must not be exceeded either for $p'$ (under tick size $d$) or for $p^*$ (under tick size $d/2$).

Under sufficiently large tick sizes many price-sensitive traders face prohibitively high costs of entering the market, and reducing tick size initially increases the average trade size. However, further reduction of tick size eventually leads to a reduction in the average trade size as the effect on large traders begins to dominate. Figs. 3 and 4 highlight the effect of tick size reduction on large traders and on the average trade size when tick size is sufficiently small. Overall, given the effect of changing tick size on trade sizes of different trader types, we conclude that a strictly positive tick size maximizes the average trade size in this market.

Finally, we consider the effect of changing tick size on specialist participation and profits. Specialist participation is measured as the fraction of the order flow that the specialist takes. Specialist profit is a per-trade average. Fig. 5 reveals that specialist participation rises sharply as tick size falls, because undercutting is so much easier with a smaller tick. When the tick is $\frac{1}{8}$, specialist participation is still only 3.6%, but at a tick of $\frac{1}{32}$, the specialist takes nearly half of all order flow.

Next, observe that specialist profits also rise sharply as tick size falls, again because the specialist undercuts more aggressively. Surprisingly, in practice, reducing the tick size has not led to a large rise in the price of a seat on the NYSE. Recent reductions in trading

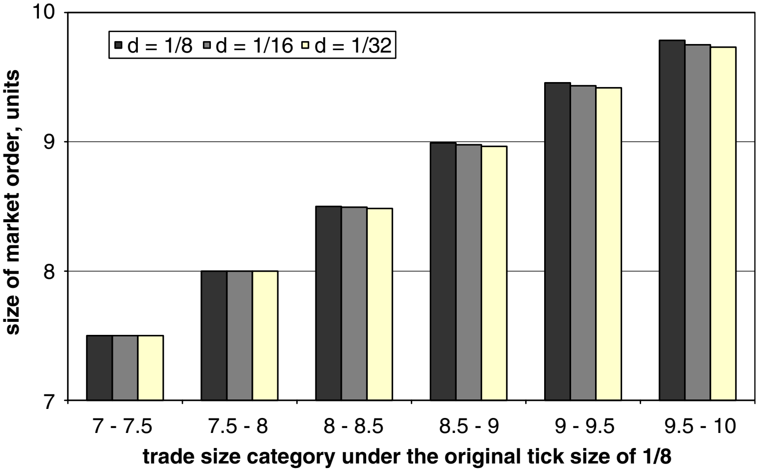


Fig. 3. The figure shows the change in the average size of market order, as tick size falls from $\frac{1}{8}$ to $\frac{1}{16}$ and then to $\frac{1}{32}$, for six groups of market order traders. Each group is defined by the size of market order that this group wishes to trade under a tick size of $\frac{1}{8}$. Thus the group that submits smaller orders under $\frac{1}{8}$ does not reduce order size, on average, as tick size goes to $\frac{1}{16}$ and then to $\frac{1}{32}$. The group that submits larger orders under $\frac{1}{8}$ does reduce order size, on average, as tick size falls. Size of market order is in units. The figure is based on numerical simulations with the following parameters: asset value $v = 1 + \eta, \eta$ small; crowd reservation price $r = 3$; price elasticity of market demand, β, is uniform on [1, 5]; liquidity shock, N, is uniform on [0,10]; per-unit cost of submitting a limit order, $c = \frac{1}{33}$.

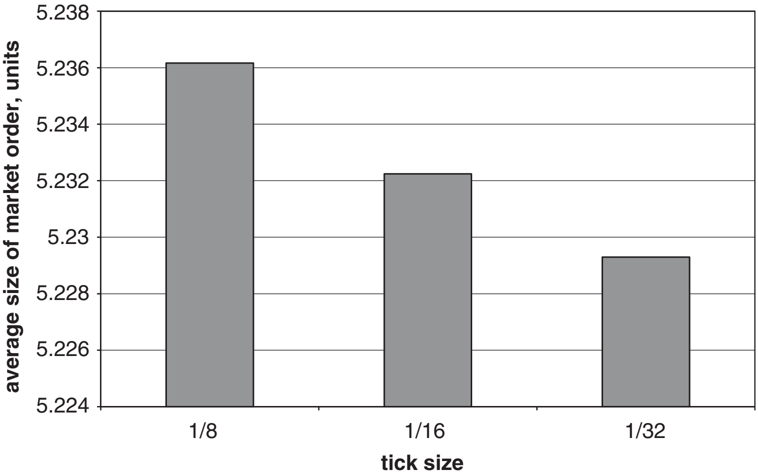


Fig. 4. The figure shows the average size of market order as tick size falls from $\frac{1}{8}$ to $\frac{1}{16}$ and then to $\frac{1}{32}$. Average size of market order is in units and is integrated across all market trader types (β, N). The figure is based on numerical simulations with the following parameters: asset value $v = 1 + \eta, \eta$ small; crowd reservation price $r = 3$; price elasticity of market demand, β, is uniform on [1, 5]; liquidity shock, N, is uniform on [0,10]; per-unit cost of submitting a limit order, $c = \frac{1}{33}$.

volume over time on the NYSE are clearly too small to explain why seat prices have not risen. Perhaps the true explanation is that the specialist's increased ability to undercut has led traders to split their orders into ever-smaller components, reducing the specialist's

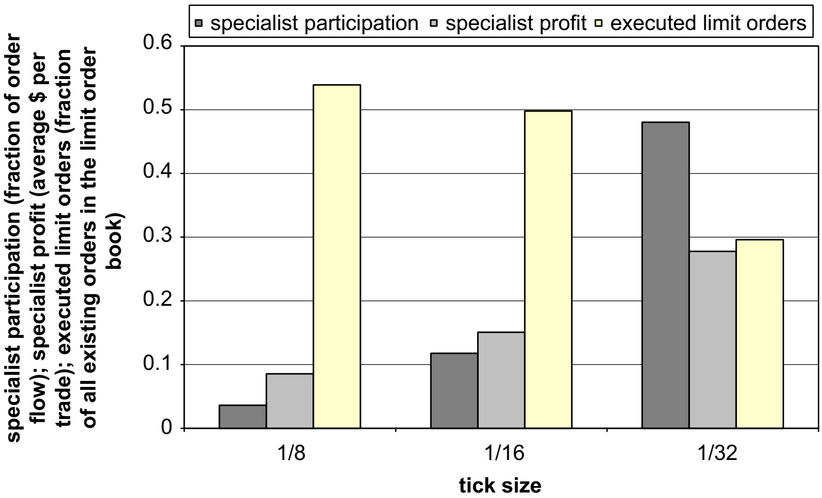


Fig. 5. The figure shows, for different values of tick size, (a) specialist participation rate, measured as a fraction of order flow; (b) specialist profit, measured as a dollar per-trade average; and (c) executed limit orders, measured as a fraction of all existing limit orders in the limit order book, up to the crowd's reservation price. The figure is based on numerical simulations with the following parameters: asset value $v = 1 + \eta, \eta$ small; crowd reservation price $r = 3$; price elasticity of market demand, β, is uniform on [1, 5]; liquidity shock, N, is uniform on [0,10]; per-unit cost of submitting a limit order, $c = \frac{1}{33}$.

advantage. The United States Government Accountability Office Report (2005) found that average trade size on the NYSE fell by 67% between 1999 and 2004. This is also consistent with the finding in Coughenour and Harris (2004) that for large stocks, specialist profits and participation have fallen with the switch to decimalization. Indeed, in our model, were all orders smaller than the aggregate depth at $p_1$, when $p_1$ is not on the grid, the specialist's share would fall to zero. Fig. 6 provides the insight into how specialist profits are sensitive to order size. It confirms that most of the increase in specialist profit comes from getting large market orders, as opposed to small orders.

## 4. Empirical implications

On a hybrid market when tick size falls so too should order size for sufficiently large orders. For smaller orders, order size might rise. This has implications for the distribution of order sizes as a function of tick size: as tick size falls average order size could rise or fall but dispersion in the distribution of order sizes should fall.

In contrast, on a pure limit order market, our model predicts that when tick size falls, cumulative depth at each price remains unchanged and trade size rises. To see this, note that the equilibrium depths at every price in the limit order book are still determined by the zero-profit condition (6). The cumulative depths in the pure limit market are then equal to the previously defined threshold amounts $t_j$ at the corresponding prices in the hybrid market. This is because in the pure limit market, the probability of executing a marginal limit order is the probability that the market order exceeds the cumulative depth at that price; and in the hybrid market, the probability of execution is the probability that the

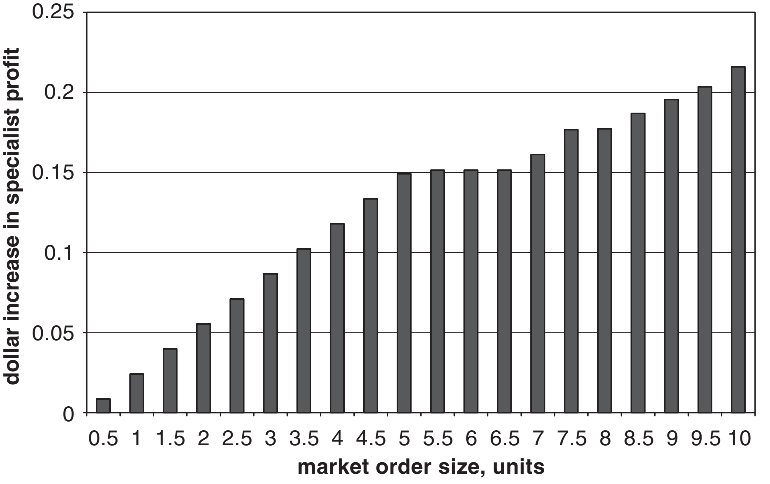


Fig. 6. The figure shows the amount of dollar increase in specialist profits as tick size is reduced from $\frac{1}{16}$ to $\frac{1}{32}$, for a given size of market order. Thus if specialist trades against a small trader, the dollar increase in the specialist profit is modest as tick size falls. However when specialist trades against a large trader, specialist profits increase substantially as tick size falls. The figure is based on numerical simulations with the following parameters: asset value $v = 1 + \eta, \eta$ small; crowd reservation price $r = 3$; price elasticity of market demand, β, is uniform on [1, 5]; liquidity shock, N, is uniform on [0,10]; per-unit cost of submitting a limit order, $c = \frac{1}{33}$.

market order exceeds the threshold amount at the same price. From (6) then, cumulative depths in a pure limit market are independent of tick size at any given price.

Now consider trade size. A market order trader whose highest affordable price is $p$ will not submit an order exceeding the cumulative depth at that price. Let $p$ be on the grid. Consider a tick size reduction from $d$ to $d/2$. Because cumulative depth at $p$ does not change, neither does market order size. Generically, the market trader's highest affordable price is not on the grid—without loss of generality, let it be a hitherto untradeable $p - d/2$. At the old tick size $d$, the market trader's order was no larger than the cumulative depth at $p - d$. Halving the tick size increases the largest affordable order size: it is now the cumulative depth at $p - d/2$. Hence, generically, trade size on a limit order market increases when tick size falls.

Some of our results are affected when market traders can split orders. Suppose that there are fixed costs to placing market orders. Glosten (1994) shows that such fixed costs are themselves barriers to order splitting. Without fixed costs, everyone would split orders so that no order would exceed the aggregate depth at the first price. The fact that all order sizes often exceed the depth at the quote suggests that fixed costs prevent order splitting to some extent. On a pure limit order market, the benefit of splitting is, if anything, reduced when tick size falls. Take as given the distribution for $N$ when tick size, $d$, is large. Reducing $d$ to $d'$ reduces trading costs for a given order size (because cumulative depths are unchanged and intermediate prices become available), so that there is less incentive to split orders than before. On a hybrid market, decreasing the tick increases trading costs only for large orders. Therefore, it is only these orders that would be more likely to split into smaller orders as the tick size fell. This again leads to the conclusion that on a hybrid market, although not a pure limit market, the dispersion of order sizes would fall with tick size.

Without order splitting, our model predicts that both specialist participation and specialist expected profits will rise as tick size falls. However, based on the model, a natural conclusion is that as tick sizes fell on the NYSE, order splitting should have become more common, leading to what has been observed, a smaller average trade size. With order splitting, specialist participation rates and profits might fall, which has also been observed, on average, on the NYSE. A cross-sectional implication is that for stocks where the smaller tick caused order splitting to become more prevalent, i.e., stocks with large average trade sizes, specialist participation rates and profits should not increase much, and might perhaps fall. In contrast, where decimalization did not lead to substantially smaller trades, specialist profits and participation rates should rise. This is an issue for further empirical investigation.

## 5. Conclusion

This paper studies the effect of changing tick size on liquidity and on the behavior of market participants in a hybrid market such as the NYSE. The general message supported by our results is that decreasing tick size too much may have undesirable effects on liquidity, trade sizes, and trading costs for some traders. In the context of price-sensitive market demand, we demonstrate that cumulative depth in a hybrid market decreases as tick size falls. We also show that for sufficiently price-sensitive market demand, when tick size is too small, equilibrium features wide quoted spreads, very little trading activity, and an empty limit order book. Finally, we demonstrate via a numerical example an intuitive result that different types of traders disagree on the optimal tick size: traders who submit smaller orders prefer smaller tick sizes.

## Appendix

**Proof to Proposition 1.** Seppi (1997) shows uniqueness when the second term on left-hand side of (6) is equal to one. Here, for each distribution of $\beta$, and given the price grid, we get a unique value for the second term. Hence, we still get a unique value for the first term on the left-hand side. Hence the solution is unique even in the price-sensitive case, provided that the equilibrium exists. □

**Proof to Proposition 2.** See Seppi (1997), Proposition 8. The validity of the proof does not depend on the price sensitivity of the market order. It does, however, depend on the condition that the threshold amount $t_j$ be independent of tick size and be monotonically increasing in price $p_j$. We show later that, with sufficiently price-sensitive market orders, the monotonicity condition is violated for a small enough tick size. □

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